Understanding Coefficient of Power (Cp) and Betz Limit

The coefficient of power of a wind turbine is a measurement of how efficiently the wind turbine converts the energy in the wind into electricity.

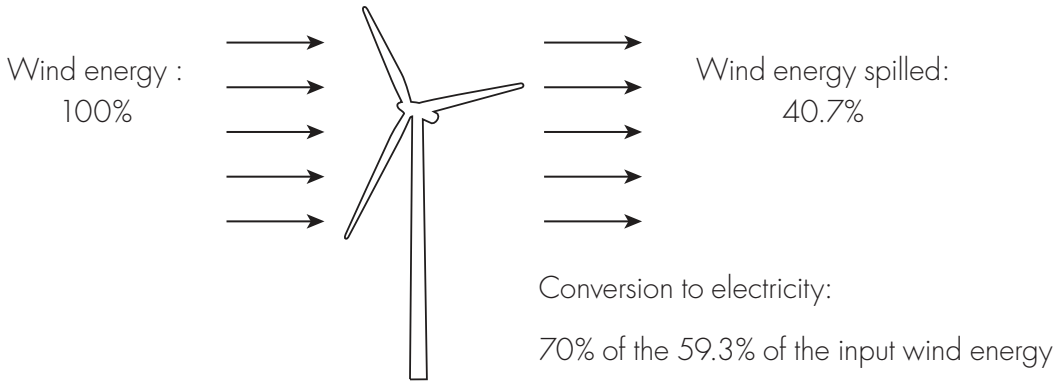
By now you already know how to calculate the amount of electricity a wind turbine is producing, and you also know how to calculate the total power available in a given area of wind. To find the coefficient of power at a given wind speed, all you have to do is divide the electricity produced by the total energy available in the wind at that speed.

$$Cp = \frac{\text{Electricity produced by wind turbine}}{\text{Total Energy available in the wind}}$$

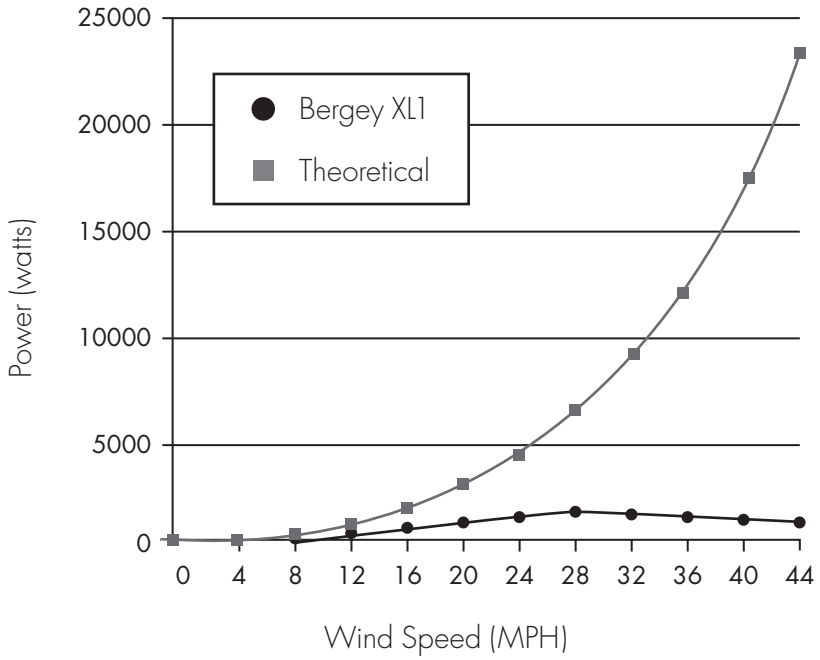
Wind turbines extract energy by slowing down the wind. For a wind turbine to be 100% efficient it would need to stop 100% of the wind - but then the rotor would have to be a solid disk and it would not turn and no kinetic energy would be converted. On the other extreme, if you had a wind turbine with just one rotor blade, most of the wind passing through the area swept by the turbine blade would miss the blade completely and so the kinetic energy would be kept by the wind.

Betz Limit

Albert Betz was a German physicist who calculated that no wind turbine could convert more than 59.3% of the kinetic energy of the wind into mechanical energy turning a rotor. This is known as the Betz Limit, and is the theoretical maximum coefficient of power for any wind turbine.



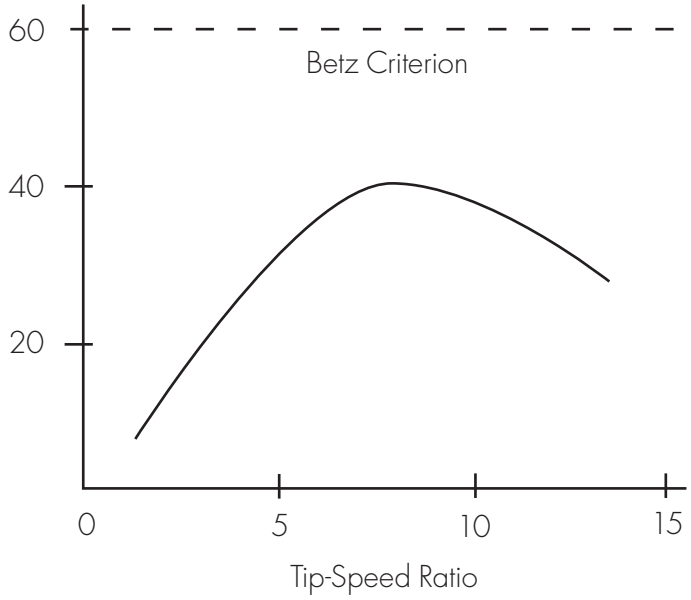
In the diagram shown above, the wind turbine converts 70% of the Betz Limit into electricity. Therefore, the Cp of this wind turbine would be 0.7 x 0.59 = 0.41. So this wind turbine converts 41% of the available wind energy into electricity. This is actually a pretty good coefficient of power. Good wind turbines generally fall in the 35-45% range.



This graph shows two power curves (1). The graph shows the actual power produced at various wind speeds by a Bergey XL1 (1 kW rating) wind turbine. It also shows the theoretical power in the wind at these wind speeds. When the wind blows at 28 mph, the Bergey turbine produces about 1,200 Watts. At the same wind speed, you can see that there is theoretically about 6,000 Watts of power in the wind. So, to find the coefficient of power for the Bergey, divide 1,200 by 6,000. The Bergey XL1 has a Cp of about 0.2 or 20% at 28 mph winds.

Though this Cp is pretty low, the fact is that small scale (1-100 kW) always have lower efficiencies than large scale wind turbines. Why do you think small scale wind turbines would be less efficient? Knowing this, how efficient do you think a well-designed Kidwind model turbine is?

Remember that the Tip Speed Ratio of a wind turbine is an essential factor to how efficient that turbine will perform (2). This graph to the right shows the relationship between tip-speed ratio (TSR) and the coefficient of power (Cp).



SAMPLE PROBLEMS

1. A 50 kW wind turbine operates at peak efficiency when the wind blows at 26 miles per hour. At this rate, it reaches 62% of the Betz limit. What is the coefficient of power for this wind turbine?

2. Another wind turbine produces 50 volts and 20 amperes at a certain wind speed. You measured the theoretical power in the wind at this speed to be about 3,900 watts. What is the coefficient of power for this turbine?

3. a. You made a homemade wind turbine that has 3 blades that are one meter long each. You live at sea level so the air density is about 1.23 kilograms per meter cubed. The wind is blowing at 12 meters per second. What is the theoretical power output in this wind?

b. You hook your turbine up to a multimeter and find that it is pumping out 12 amps and 33 volts. What is your actual power output?

c. Now calculate the coefficient of power for the turbine.

4. A man in a fine Italian suit comes to your door offering to sell you a wind turbine to go on your roof. He says that you will be able to produce all of the electricity required for your home because his "WindMaster 5,000" turbine has reached an efficiency of 73%. How would you respond to this salesman? Would you buy this turbine?

5. You ask the salesman what the Tip Speed Ratio of his wind turbine is and he responds: "Uhh…it's really great! Usually around 25 or so!" Based on the graph on the previous page about Tip Speed Ratio and power coefficient, do you think this would be a "great" tip speed ratio? Why does the efficiency seem to go down with higher TSRs?